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A supply chain model of vendor managed inventory

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Abstract

This research evaluates how vendor managed inventory (VMI) affects a supply channel. Specifically, VMI always leads to a higher buyer's profit, but supplier's profit varies. In the short-term, VMI is found to reduce total costs of the channel system, but under certain cost conditions between buyer and supplier, it could decrease the purchasing price and supplier's profit. In the long-run, it could more likely increase supplier's profit than in the short-run. Finally, VMI is an effective supply chain strategy that can realize many of the benefits obtainable only in a fully integrated supply chain. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Vendor managed inventory (VMI), also known as consignment inventory on other occasions, has been widely used in various industries. For instance, one survey found that in hospital materials management, VMI achieved higher penetration than just-in-time and stockless methods (Gerber, 1991). Another survey of the mass retail industry indicated that VMI programs would multiply in the next few years (Andel, 1996). Major retailers such as Wal-Mart, Kmart, Dillard Department Stores, and JCPenney are among the earlier adopters of VMI. Telecommunications giants such as Lucent Technologies are in the process of converting much of their materials

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management systems to VMI. This recent popularity of VMI has led to the claim that vendor managed inventory is the wave of the future and the concept will revolutionize the distribution channel (Burke, 1996; Cottrill, 1997). Whether such a trend will persist depends on the economic benefits brought to the members along a supply chain by a VMI program. There is, however, lack of in-depth economic analysis of such a program, especially in terms of its impacts on the parties involved. For instance, while consignment in the health care industry has become more and more popular in recent years, confusion exists as to when and why consignment is effective.

There is evidence that VMI is beneficial to both a buying company and a supplying company, though the supplier may take a longer period of adjustment and reconfiguration before the benefits of VMI can be realized. Northwestern Steel and Wire as a supplier, reported by Nolan (1998), was able to generate more volume after VMI because inventory turns were up. Northwestern's customers placed more orders with Northwestern after seeing real benefits and efficiency gains with the VMI programs.

On the other hand, some parties involved in a VMI relationship, suppliers in particular, are less certain about these potential benefits and tend to accept VMI as a necessity due to intense global competition. For instance, Air Products and Chemicals (AP&C) is faced with the dilemma that its customers want zero inventory, thereby tying up more of AP&C's working capital. The company does not believe this would improve business process but rather would increase the total administration and processing costs (Gamble, 1994).

A VMI-consignment is essentially an arrangement whereby the owners of goods, the "consignor", delivers its goods to another party, the "consignee", for use or for sale by the consignee, with the proceeds of the sale being remitted to the consignor only after the actual use/sale (Fagel, 1996). A typical VMI program involves a supplier which monitors inventory levels at its customer's warehouses and assumes responsibility for replenishing that inventory to achieve specified targets through the use of highly automated electronic messaging systems (Copacino, 1993). The supplier thus makes the replenishment decision, rather than waiting for the customer to reorder the product. Anecdotal evidence suggests that a consignee may enjoy reductions in holding costs and some operational costs plus cash flow benefits (e.g., Benefield, 1987), while a consignor needs to bear the burden (of inventory carrying and demand forecasting) but probably gain chances to improve other production and marketing efficiency (Cottrill, 1997). Thus a systematic evaluation of the profit implications of a VMI program on both trading partners will certainly facilitate future supply chain coordination.

One aspect of VMI, information sharing, has recently been addressed. It is believed that since the parties involved share sales information under VMI, less information distortion should be expected (Lee et al., 1997; Chen et al., 2000). As a consequence, inventory and other production costs will likely be reduced while capacity utilization will be increased, as demonstrated by Xu et al. (2001) and Waller et al. (1999). There is also a large set of literature written around the theme of channel coordination. Thomas and Griffin (1996) provide a comprehensive review of the topic. A few notable studies related to buyer–vendor coordination include Lee and Rosenblatt (1986), Banerjee (1986), Anupindi and Akella (1993), Kohli and Park (1994), and Weng (1995) which analyze the separate or joint optimal ordering policies and/or the optimal pricing/quantity discount schedules. Hung et al. (1995) develop a simple inventory control method for a consignment system for determining delivery period and safety stock level in the COCK system of Philips (Taiwan). However, no studies have examined the optimal ordering and pricing schemes under a

VMI program. The impact of VMI on both parties involved, buyer and vendor, has not been evaluated either.

The purpose of this paper is to evaluate the short-term and long-term impact of VMI on supply chain profitability by analyzing the inventory systems of the parties involved. The impact of VMI is also compared with that of full channel coordination. It is found that in the short-term VMI can accomplish what full channel coordination is set to accomplish. We formulate appropriate mathematical models for a buyer–supplier channel structure, examine the effects of a VMI strategy on the various cost components of both parties, and then analyze the role of VMI in a supply chain initiative. In particular, the effects of an integrative VMI program on total relevant costs and profits will be investigated.

2. Model structure

We investigate vendor managed inventory issues using a model based on supply chain relationships with a focus on inventory systems, purchase prices and purchase quantities. The model contains two parties along a supply chain: a buyer and a supplier (e.g., an OEM supplier). The buyer company purchases its final product (or a major component of it) from the OEM supplier. Thus the final product sales quantity of the buyer is the same as the purchase quantity or directly proportional to it. The buyer company appears to be the "leader" in this relationship, in that it specifies order quantity according to its own cost characteristics, and determines purchase prices for certain amounts of products provided by the supplier. On the other hand, the supplier has no choice but accepts the prices. The quantity the supplier is willing to provide is determined by the supplier for a given purchase price.

Several assumptions commonly used in inventory-channel coordination research (see, e.g., Kohli and Park, 1994; Weng, 1995) are made to facilitate the analysis of the consignment issue. First, the inventory system of the buyer can be described by an economic order quantity (EOQ) policy based on deterministic demand, no stock-outs and deterministic lead-times. The widely used EOQ policy is considered here due to its relative robustness in a variety of situations (Lowe and Schwarz, 1983; Axsater, 1996), and due to the fact that VMI items observed in practice are often the more popular, standard ones with predictable use (Lamb, 1997; Nolan, 1997). Let y denote the annual demand (i.e., annual sales rate and purchase quantity) for the final product of the buyer.

Incorporating the demand function for the final product of the buyer, the profit function of the buyer is given as follows:

$$\Pi_{\mathrm{B}} = p(y)y - wy - \left(\frac{s_{\mathrm{B}}y}{Q^{\mathrm{B}}} + \frac{h_{\mathrm{B}}}{2}Q_{\mathrm{B}}\right), \tag{1}$$

where $p(y)$ is the reversed demand function (sale price) of the final product, which is decreasing in y; w is the contract purchase price determined by the buyer; h_{B} and s_{B} are the buyer's inventory carrying cost/unit and order setup cost, respectively.

The buyer's order quantity in this case is $Q_{\mathrm{B}} = \mathrm{EOQ}_{\mathrm{B}} = (2s_{\mathrm{B}}y/h_{\mathrm{B}})^{1/2}$ and the buyer's inventory ordering and holding costs $\mathrm{INV}_{\mathrm{B}} = (2h_{\mathrm{B}}s_{\mathrm{B}}y)^{1/2}$. Thus,

$$\Pi_{\mathrm{B}} = p(y)y - wy - (2h_{\mathrm{B}}s_{\mathrm{B}}y)^{1/2} \tag{1'}$$

Since the supplier takes the order quantity from the buyer as given ($Q_{\mathrm{B}} = \mathrm{EOQ}_{\mathrm{B}}$) and makes the necessary delivery, the supplier's profit function, after accounting for order set up, inventory holding, and production and distribution costs, is demonstrated as follows:

$$\begin{aligned}\Pi_{\mathrm{S}} &= wy - c(y) - \left(\frac{s_{\mathrm{S}}y}{Q_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{2}Q_{\mathrm{B}}\right) \\ &= wy - c(y) - \left(\frac{h_{\mathrm{B}}s_{\mathrm{B}}y}{2}\right)^{1/2}\left(\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right),\end{aligned} \tag{2}$$

where $c(y)$ is the other production and distribution cost of the supplier, $c'(y) > 0$ and $c''(y) > 0$; h_{S} and s_{S} represent the supplier's revised carrying and order setup costs, respectively. Note that the supplier's lot size is treated as an integer multiple (m) of that of the buyer's; h_{S} and s_{S} are calculated as: $h_{\mathrm{S}} = (m-1)h'_{\mathrm{S}}$ and $s_{\mathrm{S}} = s_{\mathrm{P}} + s'_{\mathrm{S}}/m$, where $h'_{\mathrm{S}}, s_{\mathrm{P}}$ and s'_{S} are, respectively, the supplier's "actual" unit inventory holding cost, fixed order processing cost per buyer's order, and setup (ordering) cost per supplier's acquiring order. See Joglekar (1988) and Weng (1995) for more details.

For any given purchase price w from the buyer, the supplier chooses a quantity y to maximize its profit and it can be obtained from the first-order condition:

$$w = c'(y) + \frac{1}{2}\left(\frac{h_{\mathrm{B}}s_{\mathrm{B}}}{2y}\right)^{1/2}\left(\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right). \tag{3}$$

Realizing this relationship between purchase price and the quantity the supplier is willing to provide, the buyer then maximizes its profit by choosing the optimal quantity y^*, such that:

$$p'(y^*)y^* + p(y^*) - c'(y^*) - c''(y^*)y^* - \frac{1}{4}\left(\frac{h_{\mathrm{B}}s_{\mathrm{B}}}{2y^*}\right)^{1/2}\left(\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right) - \frac{1}{2}\left(\frac{2h_{\mathrm{B}}s_{\mathrm{B}}}{y^*}\right)^{1/2} = 0 \tag{4}$$

or

$$p'(y^*)y^* + p(y^*) - c'(y^*) - c''(y^*)y^* - \frac{1}{4}(w^* - c'(y^*)) - \frac{1}{2}\left(\frac{2h_{\mathrm{B}}s_{\mathrm{B}}}{y^*}\right)^{1/2} = 0. \tag{4'}$$

y^* exists when the second-order condition is satisfied (which is assumed here) and can be solved numerically with relevant demand and cost function forms. Therefore, the exact order of events is that the buyer, using knowledge of the supplier's cost and willingness to supply at a given purchase price, will offer the supplier a price so as to get the optimal quantity the buyer desires.

Now the two parties decide to adopt a VMI system, e.g., the buyer no longer manages its inventory system and leaves it to the supplier to determine inventory levels, order quantities, lead times, etc. As a result, the supplier now has the combined inventory with order setup cost ($s_{\mathrm{S}} + s_{\mathrm{B}}$) and carrying cost ($h_{\mathrm{S}} + h_{\mathrm{B}}$). The supplier's profit function with VMI-consignment becomes

$$\Pi^c_{\mathrm{S}} = w_c y - c(y) - [2(s_{\mathrm{S}} + s_{\mathrm{B}})(h_{\mathrm{S}} + h_{\mathrm{B}})y]^{1/2}, \tag{5}$$

where w_c represents the new pricing method the buyer uses to induce the supplier to manage the buyer's inventory system. That is, the contract purchase price under VMI is w_c. Please note that the subscript c is used to index decision variables, such as purchase prices and quantities, and the superscript c is used to index profits in the VMI-consignment system hereafter.

The buyer's profit function becomes:

$$\Pi_{\mathrm{B}}^{c} = p(y)y - w_c y. \tag{6}$$

Note that the buyer does not manage its inventory system with VMI, and thus its inventory costs are no longer in the profit function.

As the supplier maximizes its profit with the buyer's inventory added into the supplier's cost and profit functions, the following relationship between purchase price and purchase quantity can be obtained from the first-order condition of supplier's profit function:

$$w_c = c'(y_c) + \frac{1}{2}\left(\frac{2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{B}} + s_{\mathrm{S}})}{y_c}\right)^{1/2}. \tag{7}$$

Again, the optimal purchase quantity can be obtained from the first-order condition of buyer's profit function (6) by incorporating (7):

$$p'(y_c^*)y_c^* + p(y_c^*) - c'(y_c^*) - c''(y_c^*)y_c^* - \frac{1}{4}\left(\frac{2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{B}} + s_{\mathrm{S}})}{y_c^*}\right)^{1/2} = 0, \tag{8}$$

or

$$p'(y_c^*)y_c^* + p(y_c^*) - c'(y_c^*) - c''(y_c^*)y_c^* - \frac{1}{4}(w_c^* - c'(y_c^*)) = 0. \tag{8'}$$

The optimal purchase price, w_c^*, and final product sales price, p_c^*, can be calculated respectively based on y_c^*. For both of the above optimization problems, with and without VMI, the second-order conditions are investigated and assumed to have negative signs.

Another relevant concept in this paper is the extent of channel profit, as the sum of (1) and (2):

$$\Pi_{\mathrm{B}} + \Pi_{\mathrm{S}} = p(y)y - c(y) - \left(\frac{s_{\mathrm{B}}y}{Q_{\mathrm{B}}} + \frac{h_{\mathrm{B}}}{2}Q_{\mathrm{B}}\right) - \left(\frac{s_{\mathrm{S}}y}{Q_{\mathrm{S}}} + \frac{h_{\mathrm{S}}}{2}Q_{\mathrm{S}}\right). \tag{9}$$

Schenck and McInerney (1998) note that to assess results in a VMI relationship with a retailer, it is important to consider the VMI impact on not just retail, but total supply chain as well. The independent actions of each party in the supply chain for their own goals frequently are not conducive to the promotion of the common goal of the whole supply chain. Below, after we evaluate the impact of VMI on each party's profit position, we also attempt to assess its effect towards the goal of supply chain profit maximization.

3. The short-term motivation of VMI

VMI leads to immediate changes in both buyer's and supplier's inventory management, which can be considered as the direct, or *short-term*, effects of VMI implementation. At the early stage of VMI, sales and purchase quantities are relatively stable due to the market constraints and other contractual or public agreements with other parties. For this reason, the short-term effects will be evaluated *to the extent that the annual purchase quantity y is the same as before*. Purchase price, however, can be adjusted relatively quickly as a new VMI contract is negotiated and determined. Arguably, the direct effects of VMI on firm performance such as costs and profits may be the initial incentive and benefits for the firms adopting VMI or the major concerns for some others not to do so. As observed in many examples noted within this paper, many companies seemed to start or refute a VMI program based on its expected direct effects on their companies.

3.1. The inventory-related cost perspective

For VMI to be considered and accepted by both parties, it has to be able to induce some observable benefits, e.g., help reduce inventory-related costs. VMI's direct benefits to the buyer's side are straightforward and have been documented in practice, while those to the suppliers are more diverse and controversial. Although some other strategic or managerial considerations, such as strengthening competitive advantage, tightening buyer–supplier relationship or partnership or simply surviving, might play a role in the supplier's decision to adopt VMI, the bottom line is whether or not VMI could eventually save costs or generate revenues for the supplier. We will show that, from the perspective of cost savings, VMI allows the companies to cut total inventory-related costs and thus provides a strong incentive for both firms to integrate their inventory systems.

Lemma 1. *In the short-term, VMI will reduce the total inventory-related cost of the whole system (buyer and supplier together).*

Proof. Please see Appendix A.

Cross (1993) argues that one of the most successful endeavors to cut supply chain costs in an attempt to boost profitability is continuous replenishment planning, also known as VMI in their specific case. The above result gives a reason to such an argument and illustrates that managing the entire inventory system by one of the parties allows the supply chain to be better synchronized according to both parties' cost characteristics. As long as the two individual inventory systems are not identical, or $s_S/s_B \neq h_S/h_B$, the system is not perfectly matched and there exists room to improve the overall performance by integrating the operations of two parties. This is good news as both trading partners are under tremendous pressure to get rid of waste and inefficiency and build better business processes (Gamble, 1994).

The reduction of combined inventory-related costs through VMI, however, does not necessarily imply a cost reduction in the supplier's inventory system, although zero inventories can be realized on the buyer side. Rather, since the supplier handles the combined inventory system, the supplier's inventory costs under VMI are likely to increase. There is, however, a possibility that the supplier's inventory-related cost with VMI is less than that without VMI, as given by the following Lemma.

Lemma 2. *In the short-term, VMI will increase the supplier's inventory-related costs under a "normal" condition, if and only if*

$$\left\{2-\left[\left(1+\frac{s_S}{s_B}\right)^{1/2}-\left(1+\frac{h_S}{h_B}\right)^{1/2}\right]^2\right\}\geqslant 0. \tag{10}$$

Proof. Please see Appendix A.

From Lemma 2, VMI may possibly reduce the combined inventory-related cost of the supplier so much that it is even lower than the supplier's previous inventory-related cost without VMI, if the above normal condition is not satisfied.

From the above condition, a significant mismatch between the buyer's and supplier's inventory systems creates the potential for the substantial *direct* cost savings when introducing VMI. An example of such significant mismatch is $h_S = h_B$ while $s_S > 7s_B$, i.e., the order setup cost at the supplier is 7 times or more of that at the buyer, indicating that the EOQ_S at the supplier is $7^{1/2} = 2.646$ times or more of the EOQ_B at the buyer. Naturally this kind of scenario is where a VMI strategy is most powerful, as commonly observed in practice (Harrington, 1996). In this kind of situation with a significant cost mismatch, the independent ordering practice of either the buyer or the supplier, based on their own individual goals tends to result in much higher total costs for the other party involved and far less optimal system performance. Therefore, taking over the buyer's inventory would allow the supplier to adjust order sizes based on the overall system conditions and possibly reduce the inventory-related cost of the whole system dramatically.

The structure of many inventory systems in the supply chains, however, may not fit into this rather "strong" condition that is favorable to suppliers. As observed, suppliers' lack of enthusiasm has been reported as one of the important reasons why VMI has not been adopted throughout the supply chains. In many cases, the inventory-related costs may increase on the supplier side as a result of VMI. This normally happens when the mismatch between the two inventory systems is not so overwhelming as it is in the case above. We will focus on this normal situation as indicated by condition (10) in our discussions hereafter.

3.2. The purchase price and profit perspective

Now let us examine the impact of VMI on purchase price. Typically with the institution of VMI, the buyer and the seller will renegotiate a new transaction price (Speed, 1998). Intuitively, since the buyer's inventory is merged into the supplier's with VMI, the buyer eliminates its inventory-related cost while the supplier has to pick up this transferred cost. This might require the buyer to compensate the supplier for its possibly increased inventory cost. However, as we have just shown, the integrated inventory system, as a result of VMI, could yield such a low inventory-related cost that this cost is even less than the supplier's previous inventory-related cost without VMI. If this is the case, the supplier might even be willing to accept a cut on the purchase price so that the two companies can institute a VMI program.

Lemma 3. *In the short-term, VMI increases contract purchase price in the normal situation indicated by*

$$\mathrm{INV}_c \geqslant \mathrm{INV}_\mathrm{S} \text{ if and only if } \left\{ 2 - \left[\left(1 + \frac{s_\mathrm{S}}{s_\mathrm{B}}\right)^{1/2} - \left(1 + \frac{h_\mathrm{S}}{h_\mathrm{B}}\right)^{1/2} \right]^2 \right\} \geqslant 0 \tag{11}$$

and decreases it otherwise.

Proof. Please see Appendix A.

This indicates that if the combined inventory-related costs after VMI is adopted are no less than the supplier's previous inventory-related costs without VMI (the normal case), the contract purchase price would increase after VMI. The implications are that in the cases when VMI increases the inventory-related costs of the supplier, the purchase price has to be raised by the buyer as an effort to compensate the supplier's possible losses.

This increase in purchase price *in the normal case*, however, will not guarantee that the supplier will be able to achieve higher level of profits. As a matter of fact, as can be seen in the following proposition, VMI could result in less profit for the supplier and more profit for the buyer.

Proposition 1. *In the short-term, the buyer's profit level will always be increased after VMI. The supplier's profit could be decreased (for the normal case with the condition identified by Lemma 2), even with a higher purchase price offered by the buyer to compensate the increased costs due to VMI.*

Proof. Please see Appendix A.

Interestingly, the supplier's profit and the buyer's purchase price are headed towards opposite directions. Normally, when the supplier's inventory cost with VMI is not lower than its cost before adopting VMI, the contract purchase price will be higher but not yet sufficient to cover the supplier's cost increase with VMI; thereby the supplier's profit will be lower.

This proposition demonstrates the fact that because of the advantageous position in the buyer–supplier relationship, the buyer is able to extract benefits from VMI. However, even though VMI is proved to have the ability to reduce total inventory-related cost, the supplier may not benefit from this cost reduction. The buyer takes the biggest chunk of the cost savings. This whole series of results is all based on the condition that *in the short-term, the buyer insists on maintaining the previous annual purchase quantity y with the direct implementation of VMI.* This condition limits the supplier's ability to reach its optimal profit level in a longer term. In fact, the long-term relationship is built upon the supplier's adjustment in purchase quantity so that it can maximize its profit given a purchase price. Meanwhile over the long-term, the buyer could also take advantage of the opportunities provided by the lower inventory-related cost to attract more customer demand.

4. The long-term purchase quantity and profit changes under VMI

4.1. The change of optimal purchase quantity under VMI

Over a longer period of time after VMI is implemented, the indirect effects of VMI, such as changes in purchase quantity, will be observed. It has been reported in practice that companies with VMI had expected an increase in sales eventually, which would also bring up purchase quantity (Gamble, 1994; Andel, 1996; Nolan, 1998). Such expectation is indicated in Lemma 4 and Fig. 1.

Lemma 4. *In the long-run, the purchase quantity with VMI is higher than that without VMI.*

Proof. Please see Appendix A.

Annual sales volume and purchase quantity increase because VMI immediately brings about lower inventory cost in the supply chain, which allows the buyer to sell more products at a lower price.

Fig. 1 shows the generic relationships between marginal revenue (MR) and marginal costs (MC) of the buyer. It also demonstrates possible changes in quantity and price with VMI. In the buyer's decision-making process, the supplier's response has been considered. Since the total cost of the supplier, consisting of inventory-related costs and other costs ($c(y)$), is still convex (by assumption), VMI will result in a shift of the marginal cost curve to the right and the purchase quantity increases.

4.2. The long-term profit changes under VMI

Proposition 2 shows the long-term impact of VMI on profit levels of channel members, which is investigated by comparing the profit levels with and without VMI, taking into account the optimal purchase quantity change.

Proposition 2. *In the long-run under VMI, the buyer's profit will increase; and the supplier's profit may increase under the following sufficient condition:*

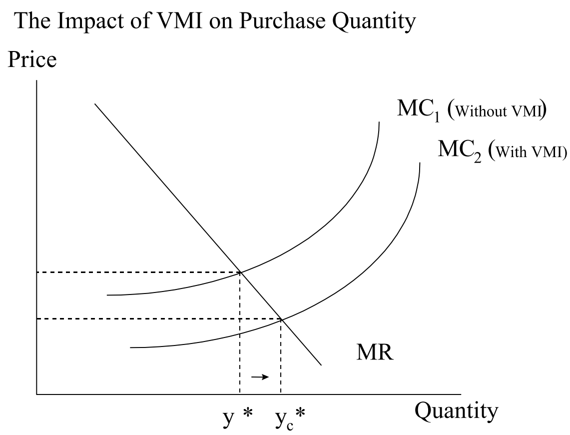

Fig. 1. Optimal buyer's purchase quantities with and without VMI.

$$\left(\frac{y_c^*}{y^*}\right)^{1/2} < 1 - \frac{2 - \left\{(1+(s_S/s_B))^{1/2} - (1+(h_S/h_B))^{1/2}\right\}^2}{2(1+(s_S/s_B))^{1/2}(1+(h_S/h_B))^{1/2}} + \frac{[c'(y_c^*) - c'(y^*)](2y^*)^{1/2}}{(h_S+h_B)^{1/2}(s_S+s_B)^{1/2}}. \quad (12)$$

Proof. Please see Appendix A.

This proposition indicates that a VMI inventory program will bring positive change to the buyer's profit, which is consistent with its short-term motivation. The long-term impact of VMI on the supplier's bottom line is improved from that of the short-run. This stems from the fact that in the long-run VMI provides solid efficiency gain for the buyer which can lower his final market sale price and obtain a higher sales volume. The higher sales volume of the buyer is then relayed to the supplier in the form of a higher purchase volume, accompanying with similar or even higher purchase price. All these help the supplier to improve his operations and profit position relatively to the short-term case. It is important to note that while the supplier's profit is reduced under the condition in Lemma 2 in the short-run, its profit can be increased under the same condition in the long-run. This result may explain why the supplier is initially only lukewarm or even suspicious of such program (Cottrill, 1997) but gradually accepts it due to other strategic considerations, opportunities to reduce other costs and to increase long-term profit.

5. Channel profit: VMI vs. full channel coordination

5.1. The short-term effects of inventory program change

As discussed in Section 2, we are equally concerned with the channel profit as it represents the effectiveness of a supply chain. First of all, the difference of joint (channel) profit before and after a VMI program can be easily shown by:

$$[\Pi_B^c(y) + \Pi_S^c(y)] - [\Pi_B(y) + \Pi_S(y)] = \left[\frac{s_B h_B y}{2}\right]^{1/2}\left[\left(1+\frac{s_S}{s_B}\right)^{1/2} - \left(1+\frac{h_S}{h_B}\right)^{1/2}\right]^2 \geqslant 0. \quad (13)$$

Note that y is added in the profit function above to highlight that the same purchase quantity is maintained before and *shortly* after the VMI implementation. This result indicates that as long as there is any mismatch between the setup cost ratio and the unit holding cost ratio of the supplier and the buyer, the total channel profit will improve with a VMI program. This is true regardless whether the supplier's profit is increased or not with a VMI program.

Second, full channel coordination is an ideal state in the management of a supply chain. It can be reached if the joint (channel) profit in (9) is maximized. Denote this optimal situation of joint profit with a superscript J. During an inventory program change to full coordination, to the extent that the purchase quantity y is held fixed in the short-term while Q_B can be adjusted by both parties, it can be easily shown from (9) that the joint profit can be maximized when

$$Q_B^J = Q_B^c = [2(s_S + s_B)y/(h_S + h_B)]^{1/2}.$$

This leads to

$$\Pi^c_{\mathrm{B}}(y) + \Pi^c_{\mathrm{S}}(y) = \Pi^{\mathrm{J}}_{\mathrm{B}}(y) + \Pi^{\mathrm{J}}_{\mathrm{S}}(y) \geqslant \Pi_{\mathrm{B}}(y) + \Pi_{\mathrm{S}}(y). \tag{14}$$

In other words, in terms of channel profit, a VMI program could reach the same direct effect in the short-term as full channel coordination. This strong result is due to the fact that we focus on the inventory systems of the two parties while holding the purchase quantity unchanged in the short-run.

5.2. The long-term effects of program change

Proposition 3. *VMI program in the long-run will lead to a channel profit better than in the short-run, which is in turn better than before a VMI program, but may still fall short of the effect of full channel coordination,* (*i.e.*)

$$\Pi_{\mathrm{B}}(y^*) + \Pi_{\mathrm{S}}(y^*) \leqslant \Pi^c_{\mathrm{B}}(y^*) + \Pi^c_{\mathrm{S}}(y^*) \leqslant \Pi^c_{\mathrm{B}}(y^*_c) + \Pi^c_{\mathrm{S}}(y^*_c) < \Pi^{\mathrm{J}}_{\mathrm{B}}(y^*_{\mathrm{J}}) + \Pi^{\mathrm{J}}_{\mathrm{S}}(y^*_{\mathrm{J}}). \tag{15}$$

Proof. Please see Appendix A.

This result indicates that a VMI program will have the effect in raising channel profit similar to, though its magnitude may not be as large as that of, the full channel coordination in the long-run. It should be noted that full channel coordination is an ideal state in supply chain management over the long-run and is achieved typically under vertical integration and thus serves as the indication of the maximum channel profit possible. VMI may not realize the full potential of full channel coordination in the long-term because the supplier cannot make other operating decisions such as the retail pricing decision for the buyer. This prevents both parties from fully taking advantage of the possible alignments of retail market with buyer's and supplier's inventory operations. Numerical results in the next section will illustrate how a VMI program performs in raising channel profits relatively to the full channel coordination.

Summarizing results in this and previous sections, it is easy to see that the opportunities to reduce other costs are instrumental to the eventual acceptance of VMI by a supplier. For instance, a VMI program is often implemented with other operating enhancements such as more efficient ordering with better EDI capabilities (Burke, 1996; Hughes, 1996; Nolan, 1998). This translates into lower-order setup costs at the buyer's and the supplier's locations, i.e., lower s_{S} and/or lower s_{B} in the profit function (5) with VMI. Following a procedure analogous to Proposition 2, it could be demonstrated that this improvement would make the supplier much more likely to embrace the VMI program.

Furthermore, with VMI the supplier is not simply relieving its customer of the cost and efforts of inventory tracking. By sharing the customer's inventory and usage information, the supplier can make more cost-effective purchasing decisions for its own materials (Lamb, 1997). The benefits of VMI are made possible through the supplier having better visibility into supply chain activity (Lee et al., 1997; Xu et al., 2001).

Even if the supplier does not have immediate opportunities to reduce other costs but may incur higher total costs in a VMI relationship, the buyer can induce the supplier to adopt the VMI program by instituting a purchase price premium beyond those indicated by (7). For instance, the purchase price premium could be set in such a way that the supplier in VMI could achieve a profit level similar to that before VMI. The buyer should have enough incentive to share its profit gain from VMI, as VMI will lead to higher channel profit and the sharing will guarantee the VMI program to be mutually beneficial to both parties. As observed in the case of Western Publishing, gain-sharing is one key factor to their successful VMI relationships with retailers such as Wal-Mart (Andel, 1996).

The advantages of VMI presented in this paper and observed in practice are convincing evidence why VMI is the preferred route to more efficient supply chain management in many companies. The program not only achieves higher level of integration than a traditional system, but also becomes a stepping stone to the ideal state of full channel coordination.

6. A specific case under linear demand

We assume the demand function can be represented by a simple linear relationship between price and quantity: $p(y) = a - by$ and $a, b > 0$. In addition, the cost function of the supplier is assumed to be: $c(y) = \delta y + 0.5\theta y^2$ and $\delta, \theta > 0$. Please note that $c'(y)$ and $c''(y) > 0$, and $p'(y) < 0$.

The example is largely based on the information from a consignment item in a telecommunications equipment manufacturer (as a supplier) and its client. In their specific case, the manufacturer's order setup cost $s_S = 1500$, which is about five times of the buyer's.

From Table 1, we observe that in the long-run, a supplier adopting a VMI program has a higher profit than its pre-VMI case when there is a significant mismatch between the supplier's and buyer's inventory cost parameters. Furthermore, from the channel profit perspective, the higher such a mismatch, the closer in profit position is the VMI coordination system is to the ideal full channel coordination. For instance, consider the maximum possible extent of channel profit improvement from pre-VMI to full channel coordination as 100%. A full implementation of VMI (i.e., in the long-run) will achieve about 77% of the maximally possible profit improvement. This convinces us that although VMI may not always reach the ideal state of full channel coordination, it can be a very effective tool of increasing total channel profit when a reasonable degree of mismatch is present in the two trading partners' inventory systems.

7. Conclusion and managerial implications

This study has focused on VMI's role as a strategy of integrated supply chain and evaluates its attractiveness to the two parties involved. It reveals that a VMI program will be effective in reducing the inventory-related costs for the system of buyer-supplier channel as a whole, even without changing any cost characteristics of the channel or demand level at the end market, a condition described as direct changeover or short-term. It achieves this through optimizing shipment quantities as observed in typical VMI practice (Hughes, 1996). The buyer will snatch most of these short-term cost savings, as its profit under VMI is always higher than that before VMI when both parties manage their respective inventory independently. Such cost savings are not always shared by the VMI supplier, because the supplier's extra burden of carrying the buyer's

Table 1
Numerical examples of profit differences before and after VMI, and relative to full channel coordination

	Long-term state's purchase quantities			Supplier's profit difference: pre-VMI vs. long-term-VMI	Channel profit: pre-VMI	Channel profit: short-term-VMI		Channel profit: long-term-VMI		Channel profit: full-channel coord.
s_S	y^*	y_c^*	y_J^*	$\Pi_S^c(y_c^*) - \Pi_S(y^*)$	$\Pi_B(y^*) + \Pi_S(y^*)$	$\Pi_B^c(y^*) + \Pi_S^c(y^*)$	Note[a]	$\Pi_B^c(y_c^*) + \Pi_S^c(y_c^*)$	Note[a]	$\Pi_B^J(y_J^*) + \Pi_S^J(y_J^*)$
150	1295	1304	1535	−1247	26,208	26,256	(6%)	26,308	(13%)	26,961
300	1290	1299	1525	−1271	25,523	25,523	(0%)	25,572	(7%)	26,190
600	1282	1292	1507	−1198	24,157	24,290	(18%)	24,342	(25%)	24,904
900	1273	1285	1492	−1027	22,798	23,248	(44%)	23,309	(50%)	23,826
1200	1264	1279	1479	−794	21,446	22,328	(62%)	22,402	(67%)	22,880
1500	**1255**	**1274**	**1467**	**−520**	**20,100**	**21,496**	**(72%)**	**21,585**	**(77%)**	**22,029**
2100	1237	1264	1445	113	17,431	20,016	(84%)	20,140	(88%)	20,526
3000	1210	1252	1417	1191	13,483	18,108	(90%)	18,291	(94%)	18,607

Note: Other demand and cost parameters are fixed at the following levels $a = 80; b = 0.01; \theta = 0.005; \delta = 40; s_B = 300; h_B = 9; h_S = 9$. Second-order conditions are carefully checked to ensure the optimality of y^*, y_c^* and y_J^*. Initially from the real world example, $s_S = 1500$. Different values of s_S are shown here to illustrate the effects of VMI under various conditions.

[a] The percentage is computed as the ratio of channel profit difference between VMI and pre-VMI relative to the channel profit difference between full channel coordination and pre-VMI.

inventory operations may not be sufficiently compensated by the VMI buyer's purchase price. A significant mismatch such as $s_S/s_B \gg h_S/h_B$, considered as a relatively "strong" condition, provides opportunities for the supplier to streamline the operations of the whole system such that its inventory-related cost is even lower than that prior to VMI, i.e., its profit higher than that prior to VMI. This clearly suggests that companies interested in adopting a VMI program should focus more on those transaction relationships where both parties' inventory systems are significantly different.

Naturally, VMI's benefits go beyond a simple switchover. In a longer period when both the buyer and the supplier adjust their production, distribution and marketing efforts to take advantage of this lower system-wide inventory-related cost, the final sales volume and thus the purchase quantity will likely increase, as observed in the logistics practice of Proctor and Gamble (Nolan, 1997). More importantly, the channel profit as a measure of the supply chain success will increase as a result, approaching a maximum scale afforded by full channel coordination and providing significant opportunities for a mutually beneficial relationship. The buyer typically enjoys a solid gain in its profit through such a long-term adjustment under VMI, while the supplier's financial gain is much less evident. However, as discussed above, other strategic benefits such as long-term partnership with the buyer and reductions in certain cost components would make the VMI program sufficiently attractive to the supplier.

One interesting implication also follows from our models when transportation costs need to be considered in a VMI relationship. In practice, very often the transportation costs of a shipment order, which are typically borne by the supplier, can be divided into a fixed cost component that does not change with the order quantity (e.g., consider a delivery where a stop at a customer's shop has to be made regardless of order quantity), and a second component that changes linearly with the order quantity. To the extent that this is true, these two cost components can be directly incorporated into the calculation of the "revised order setup cost" s_S and "revised inventory carrying cost per unit" h_S that have similar relationships with order quantity. Therefore our conclusion in this paper will remain essentially the same, with the extent of profit changes on both the supplier and the buyer to be recomputed based on the newly revised cost parameters considering the transportation operations.

8. Model limitations and future research

Just like most research, our analysis has limits, which are briefly examined here. For instance, it is possible that after the supplier takes over the buyer's inventory via VMI, the order set up cost at the buyer's location can be reduced through new ordering procedure or better communications scheme such as Electronic Data Interchange. In this case, the supplier's profit under VMI will be higher than that when there is no change in the order set up cost. It is thus even easier for both buyer and supplier to embrace VMI.

In this paper, we examine the optimal buyer–supplier behavior before and after VMI. However, many real world companies may indeed carry substantial waste and inefficiency. This suggests that improvements can be made possible from two directions: improvement through optimization under the existing non-VMI program; and improvements through switching to VMI and later through optimization under VMI. While some of the former could be examined through our model, the latter is the focus of our paper.

This paper captures the essence regarding the VMI's effect on profit, purchase price and purchase quantity where regular stock is the center of focus. Other aspects of a VMI relationship, such as better forecasting, communications and subsequent results of less safety stock and operations volatility in the supply chain, cannot be captured via this relatively simplistic model. These issues are addressed to a certain extent by Waller et al. (1999) and Xu et al. (2001), and will be empirically examined in another paper by the same authors.

Future research should also include the cases of other inventory systems, the qualitative factors in VMI implementations, and directions to improve existing VMI programs.

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Appendix A

Lemma 1. *In the short-term, VMI will reduce the total inventory-related cost of the whole system (buyer and supplier together).*

Proof. The lemma can be simply proved by comparing the total inventory-related costs of the whole system with and without VMI.

Total inventory-related cost of the system without VMI is given by summing the inventory ordering and holding costs of the buyer and the supplier:

$$\begin{aligned}\mathrm{INV} &= (2s_{\mathrm{B}}h_{\mathrm{B}}y)^{1/2} + \left(\frac{s_{\mathrm{B}}h_{\mathrm{B}}y}{2}\right)^{1/2}\left(\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right) \\ &= \frac{1}{2}(2s_{\mathrm{B}}h_{\mathrm{B}}y)^{1/2}\left[\left(1+\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}}\right) + \left(1+\frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right)\right].\end{aligned}$$

Total inventory-related cost with VMI-consignment is now borne by the supplier alone:

$$\begin{aligned}\mathrm{INV}_c &= [2(s_{\mathrm{S}}+s_{\mathrm{B}})(h_{\mathrm{S}}+h_{\mathrm{B}})y]^{1/2} \\ &= (2s_{\mathrm{B}}h_{\mathrm{B}}y)^{1/2}\left[\left(1+\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}}\right)\left(1+\frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right)\right]^{1/2}.\end{aligned}$$

Thus,

$$\mathrm{INV} - \mathrm{INV}_c = \frac{1}{2}(2s_\mathrm{B}h_\mathrm{B}y)^{1/2}\left[\left(1+\frac{s_\mathrm{S}}{s_\mathrm{B}}\right)^{1/2} - \left(1+\frac{h_\mathrm{S}}{h_\mathrm{B}}\right)^{1/2}\right]^2.$$

Comparing these inventory-related costs, one can easily verify that $\mathrm{INV} \geqslant \mathrm{INV}_c$. The equality holds when the ratio of setup costs and the ratio of the carrying costs are the same. □

Lemma 2. *In the short-term, VMI will increase the supplier's inventory-related costs under a normal condition, if and only if*

$$\left\{2 - \left[\left(1+\frac{s_\mathrm{S}}{s_\mathrm{B}}\right)^{1/2} - \left(1+\frac{h_\mathrm{S}}{h_\mathrm{B}}\right)^{1/2}\right]^2\right\} \geqslant 0.$$

Proof. This condition can be shown in the following comparison of supplier's inventory-related costs before and after VMI-consignment:

$$\mathrm{INV}_\mathrm{S} = \left(\frac{s_\mathrm{B}h_\mathrm{B}y}{2}\right)^{1/2}\left(\frac{s_\mathrm{S}}{s_\mathrm{B}} + \frac{h_\mathrm{S}}{h_\mathrm{B}}\right), \text{ and}$$

$$\mathrm{INV}_c = [2(s_\mathrm{S}+s_\mathrm{B})(h_\mathrm{S}+h_\mathrm{B})y]^{1/2} = (2s_\mathrm{B}h_\mathrm{B}y)^{1/2}\left[\left(1+\frac{s_\mathrm{S}}{s_\mathrm{B}}\right)\left(1+\frac{h_\mathrm{S}}{h_\mathrm{B}}\right)\right]^{1/2}.$$

It follows that

$$\mathrm{INV}_c \geqslant \mathrm{INV}_\mathrm{S} \text{ if and only if } \left\{2 - \left[\left(1+\frac{s_\mathrm{S}}{s_\mathrm{B}}\right)^{1/2} - \left(1+\frac{h_\mathrm{S}}{h_\mathrm{B}}\right)^{1/2}\right]^2\right\} \geqslant 0. \quad \square$$

Lemma 3. *In the short-term, VMI increases contract purchase price in the normal situation indicated by*

$$\mathrm{INV}_c \geqslant \mathrm{INV}_\mathrm{S} \text{ if and only if } \left\{2 - \left[\left(1+\frac{s_\mathrm{S}}{s_\mathrm{B}}\right)^{1/2} - \left(1+\frac{h_\mathrm{S}}{h_\mathrm{B}}\right)^{1/2}\right]^2\right\} \geqslant 0$$

and decreases it otherwise.

Proof. The lemma is proved by comparing w_c and w, the purchase prices with and without VMI. Recall that the buyer determines the appropriate contract purchase prices based on the supplier's willingness to provide certain purchase quantity, which is based on Eq. (3) without VMI and on Eq. (7) with VMI. Since the purchase quantity y is kept the same immediately following the implementation of VMI, subtracting Eq. (3) from Eq. (7) leads to

$$
\begin{aligned}
w_c - w &= \frac{1}{2}\left(\frac{2(h_B + h_S)(s_B + s_S)}{y}\right)^{1/2} - \frac{1}{2}\left(\frac{h_B s_B}{2y}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right) \\
&= \frac{1}{2}\left(\frac{h_B s_B}{2y}\right)^{1/2}\left\{\left[2\left(1 + \frac{s_S}{s_B}\right)\left(1 + \frac{h_S}{h_B}\right)\right]^{1/2} - \left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right)\right\} \\
&= \frac{1}{2}\left(\frac{h_B s_B}{2y}\right)^{1/2}\left\{2 - \left[\left(1 + \frac{s_S}{s_B}\right)^{1/2} - \left(1 + \frac{h_S}{h_B}\right)^{1/2}\right]^2\right\}.
\end{aligned}
$$

Then $w_c - w \geqslant 0$ if the normal condition in Lemma 2 is true but reverse otherwise. □

Proposition 1. *In the short-term, the buyer's profit level will always be increased after VMI. The supplier's profit could be decreased (for the normal case with the condition identified by Lemma 2), even with a higher purchase price offered by the buyer to compensate the increased costs due to VMI.*

Proof. The profit change of the supplier can be examined by comparing Eqs. (2) and (5):

$$
\begin{aligned}
\Pi_S^c - \Pi_S &= w_c y - [2(s_B + s_S)(h_B + h_S)y]^{1/2} - wy + \left(\frac{s_B h_B y}{2}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right) \\
&= (w_c - w)y - \left(\frac{s_B h_B y}{2}\right)^{1/2}\left[2\left(1 + \frac{s_S}{s_B}\right)^{1/2}\left(1 + \frac{s_S}{s_B}\right)^{1/2} - \left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right)\right].
\end{aligned}
$$

From Lemma 3, combining the difference in purchase price yields

$$
\Pi_S^c - \Pi_S = -(w_c - w)y = -\frac{1}{2}\left(\frac{h_B s_B y}{2}\right)^{1/2}\left\{2 - \left[\left(1 + \frac{s_S}{s_B}\right)^{1/2} - \left(1 + \frac{h_S}{h_B}\right)^{1/2}\right]^2\right\}.
$$

Similarly, the buyer's profit change can be obtained as

$$
\begin{aligned}
\Pi_B^c - \Pi_B &= -(w_c - w)y + (2 s_B h_B y)^{1/2} \\
&= -\frac{1}{2}\left(\frac{h_B s_B y}{2}\right)^{1/2}\left\{2 - \left[\left(1 + \frac{s_S}{s_B}\right)^{1/2} - \left(1 + \frac{h_S}{h_B}\right)^{1/2}\right]^2\right\} + 2\left(\frac{s_S h_B y}{2}\right)^{1/2} \\
&= \frac{1}{2}\left(\frac{h_B s_B y}{2}\right)^{1/2}\left\{2 + \left[\left(1 + \frac{s_S}{s_B}\right)^{1/2} - \left(1 + \frac{h_S}{h_B}\right)^{1/2}\right]^2\right\} > 0.
\end{aligned}
$$

This proves the proposition. □

Lemma 4. *In the long-run, the purchase quantity with VMI is higher than that without VMI.*

Proof. Relating Eqs. (8) and (4), the first-order conditions of the buyer's profit maximization with and without VMI, one can establish that at y^* (the optimal purchase quantity without VMI) where only Eq. (4) is satisfied,

$$p'(y^*)y^* + p(y^*) - c'(y^*) - c''(y^*)y^* - \frac{1}{4}\left(\frac{2(h_B + h_S)(s_B + s_S)}{y^*}\right)^{1/2}$$
$$\geqslant p'(y^*)y^* + p(y^*) - c'(y^*) - c''(y^*)y^* - \frac{1}{4}\left(\frac{h_B s_B}{2y^*}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right) - \frac{1}{2}\left(\frac{2h_B s_B}{y^*}\right)^{1/2} = 0.$$

Remember that the second-order conditions hold (i.e., the second-order derivatives of the buyer's profit are negative), which means the left-hand side of the above inequality is decreasing on y. In Eq. (8), y_c^* as the optimal purchase quantity after the implementation of VMI, leads the left-hand side to zero. Therefore, it can be obtained that $y_c^* \geqslant y^*$ or the purchase quantity with VMI is higher than that without VMI. □

Proposition 2. *In the long-run under VMI, the buyer's profit will increase; and the supplier's profit may increase under the following sufficient condition:*

$$\left(\frac{y_c^*}{y^*}\right)^{1/2} < 1 - \frac{2 - \left\{(1 + (s_S/s_B))^{1/2} - (1 + (h_S/h_B))^{1/2}\right\}^2}{2(1 + (s_S/s_B))^{1/2}(1 + (h_S/h_B))^{1/2}} + \frac{[c'(y_c^*) - c'(y^*)](2y^*)^{1/2}}{(h_S + h_B)^{1/2}(s_S + s_B)^{1/2}}.$$

Proof. Since y_c^* maximizes the buyer's profit in a VMI system, $\Pi_B^c(y_c^*) \geqslant \Pi_B^c(y^*)$ for any $y_c^* \neq y^*$. Comparing the optimal profit levels of the buyer with and without VMI, and using Proposition 1, we have

$$\Pi_B^c(y_c^*) - \Pi_B(y^*) \geqslant \Pi_B^c(y^*) - \Pi_B(y^*) > 0.$$

The supplier's profit change in the long-run is much less clear. Recall that $c'(y) > 0$ and $c''(y) > 0$, and note that according to Mean-Value Theorem, there exists a point y_0^* such that $y_c^* \geqslant y_0^* \geqslant y^*$ and $c(y_c^*) - c(y^*) = c'(y_0^*)(y_c^* - y^*)$. The difference of long-run profit positions of the supplier with and without a VMI program is obtained from combining Eqs. (2), (3), (5) and (7):

$$\begin{aligned}\Pi_S^c(y_c^*) - \Pi_S(y*) &= c'(y_c^*)y_c^* - c'(y^*)y^* - [c(y_c^*) - c(y^*)] - \frac{1}{2}[2(h_S + h_B)(s_S + s_B)y_c^*]^{1/2} \\ &\quad + \frac{1}{2}\left(\frac{h_B s_B y^*}{2}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right) \\ &= c'(y_c^*)y_c^* - c'(y^*)y^* - c'(y_0^*)(y_c^* - y^*) - \frac{1}{2}[2(h_S + h_B)(s_S + s_B)y_c^*]^{1/2} \\ &\quad + \frac{1}{2}\left(\frac{h_B s_B y^*}{2}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right) \\ &= [c'(y_c^*) - c'(y_0^*)]y_c^* + [c'(y_0^*) - c'(y^*)]y^* - \frac{1}{2}[2(h_S + h_B)(s_S + s_B)y_c^*]^{1/2} \\ &\quad + \frac{1}{2}\left(\frac{h_B s_B y^*}{2}\right)^{1/2}\left(\frac{s_S}{s_B} + \frac{h_S}{h_B}\right).\end{aligned}$$

Using this expression, we could establish the following sufficient condition:

$$\Pi^c_{\mathrm{S}}(y^*_c) - \Pi_{\mathrm{S}}(y^*) \geqslant [c'(y^*_c) - c'(y^*_0)]y^* + [c'(y^*_0) - c'(y^*)]y^* - \frac{1}{2}[2(h_{\mathrm{S}} + h_{\mathrm{B}})(s_{\mathrm{S}} + s_{\mathrm{B}})y^*_c]^{1/2} + \frac{1}{2}\left(\frac{h_{\mathrm{B}} s_{\mathrm{B}} y^*}{2}\right)^{1/2}\left(\frac{s_{\mathrm{S}}}{s_{\mathrm{B}}} + \frac{h_{\mathrm{S}}}{h_{\mathrm{B}}}\right) > 0,$$

if

$$\left(\frac{y^*_c}{y^*}\right)^{1/2} < 1 - \frac{2 - \left\{(1 + (s_{\mathrm{S}}/s_{\mathrm{B}}))^{1/2} - (1 + (h_{\mathrm{S}}/h_{\mathrm{B}}))^{1/2}\right\}^2}{2(1 + (s_{\mathrm{S}}/s_{\mathrm{B}}))^{1/2}(1 + (h_{\mathrm{S}}/h_{\mathrm{B}}))^{1/2}} + \frac{[c'(y^*_c) - c'(y^*)](2y^*)^{1/2}}{(h_{\mathrm{S}} + h_{\mathrm{B}})^{1/2}(s_{\mathrm{S}} + s_{\mathrm{B}})^{1/2}}. \quad \square$$

Proposition 3. *VMI program in the long-run will lead to a channel profit better than in the short-run, which is in turn better than before a VMI program, but may still fall short of the effect of full channel coordination, i.e.*,

$$\Pi_{\mathrm{B}}(y^*) + \Pi_{\mathrm{S}}(y^*) \leqslant \Pi^c_{\mathrm{B}}(y^*) + \Pi^c_{\mathrm{S}}(y^*) \leqslant \Pi^c_{\mathrm{B}}(y^*_c) + \Pi^c_{\mathrm{S}}(y^*_c) < \Pi^{\mathrm{J}}_{\mathrm{B}}(y^*_{\mathrm{J}}) + \Pi^{\mathrm{J}}_{\mathrm{S}}(y^*_{\mathrm{J}}).$$

Proof. We know from Eq. (8) that with VMI, the first order condition of the buyer's profit function is

$$p'(y^*_c)y^*_c + p(y^*_c) - c'(y^*_c) - c''(y^*_c)y^*_c - \frac{1}{4}\left(\frac{2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{B}} + s_{\mathrm{S}})}{y^*_c}\right)^{1/2} = 0.$$

While the first-order condition of joint profit optimization (full channel coordination) from Eq. (9) is

$$p'(y^*_{\mathrm{J}})y^*_{\mathrm{J}} + p(y^*_{\mathrm{J}}) - c'(y^*_{\mathrm{J}}) - \frac{1}{2}\left(\frac{2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{B}} + s_{\mathrm{S}})}{y^*_{\mathrm{J}}}\right)^{1/2} = 0, \tag{A.1}$$

where y^*_{J} is the optimal purchase quantity under full channel coordination. From the second-order condition of the supplier's profit, we have

$$-c''(y_c) + \frac{1}{4}[2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{S}} + s_{\mathrm{B}})]^{1/2} y_c^{-(3/2)} < 0.$$

Combining these two conditions, one should find

$$p'(y^*_c)y^*_c + p(y^*_c) - c'(y^*_c) - \frac{1}{2}\left(\frac{2(h_{\mathrm{B}} + h_{\mathrm{S}})(s_{\mathrm{B}} + s_{\mathrm{S}})}{y^*_c}\right)^{1/2} > 0. \tag{A.2}$$

Assume that the joint profit function is globally concave with respect to y (the second-order condition of Eq. (9) is negative), and recall that marginal profit is decreasing in y. Comparing the two first-order conditions ((A.1) with (A.2)) leads to $y^*_{\mathrm{J}} > y^*_c$. Previously in Section 4.1 we show $y^*_c \geqslant y^*$. Thus, the positive marginal channel profit at y^*_c means that as the purchase quantity increases from y^* to y^*_c the total channel profit will increase (while the marginal channel profit will decrease and becomes 0 at y_J^*). Relating to profit changes in the short-term, we can establish that in their respective long-term states,

$$\Pi_B(y^*) + \Pi_S(y^*) \leqslant \Pi_B^c(y^*) + \Pi_S^c(y^*) \leqslant \Pi_B^c(y_c^*) + \Pi_S^c(y_c^*) < \Pi_B^J(y_J^*) + \Pi_S^J(y_J^*). \quad \square$$

References

Andel, T., 1996. Manage inventory, own information. Transportation and Distribution 37 (5), 54–58.

Anupindi, R., Akella, R., 1993. Diversification under supply uncertainty. Management Science 39 (8), 944–963.

Axsater, S., 1996. Using the deterministic EOQ formula in stochastic inventory control. Management Science 42 (6), 830–834.

Benefield, D., 1987. Consignment: what the hospital CFO should know. Hospital Material Management Quarterly 8 (4), 29–37.

Banerjee, A., 1986. A joint economic lot size model for purchaser and vendor. Decision Sciences 17, 292–311.

Burke, M., 1996. It's time for vendor managed inventory. Industrial Distribution 85 (2), 90.

Chen, F., Drezner, Z., Ryan, J.K., Simchi-Levi, D., 2000. Quantifying the bullwhip effect in a simple supply chain: the impact of forecasting, lead times and information. Management Science 46 (3), 436–443.

Copacino, W.C., 1993. Logistics strategy: how to get with the program. Traffic Management 32 (8), 23–24.

Cottrill, K., 1997. Reforging the supply chain. Journal of Business Strategy 18 (6), 35–39.

Cross, G.J., 1993. Continuous replenishment planning: an untapped gold mine. Transportation and Distribution 34 (12), 49.

Fagel, A.J., 1996. Selling on consignment: another tool in the credit arsenal. Business Credit 98 (9), 6–8.

Gamble, R.H., 1994. Curse of consignment sales: carrying customers' inventory weighs down cash flow. Corporate Cash Flow 15 (10), 30–32.

Gerber, N., 1991. Objective comparisons of consignment, just-in-time, and stockless. Hospital Material Management Quarterly 13 (1), 10–17.

Harrington, L., 1996. Consignment selling: trend or Another wild idea. Transportation and Distribution 37 (6), 45–48.

Hughes, A.M., 1996. How Panduit did it. American Demographics 1996, 4–7.

Hung, J., Fun, Y., Li, C., 1995. Inventory management in consignment System. Production and Inventory Management Journal 36 (4), 1–5.

Joglekar, P.N., 1988. Comments on 'a quantity discount pricing model to increase vendor profits'. Management Science 34 (110), 1391–1398.

Kohli, R., Park, H., 1994. Coordinating buyer-seller transactions across multiple products. Management Science 40 (9), 45–50.

Lamb, M.R., 1997. Vendor managed inventory: customers like the possibilities. Metal Center News 37 (2), 42–49.

Lee, H., Padmanabhan, P., Whang, S., 1997. Information distortion in a supply chain: the bullwhip effect. Management Science 43 (4), 546–558.

Lee, H., Rosenblatt, M.J., 1986. A generalized quantity discount pricing model to increase supplier profits. Management Science 32 (9), 1177–1185.

Lowe, T.J., Schwarz, L.B., 1983. Parameter estimation for the EOQ lot-size model: minmax and expected value choices. Naval Research Logistics 30, 367–376.

Nolan, K., 1997. For Marmon/Keystone, VMI offers real procurement solutions. Metal Center News 37 (2), 50–60.

Nolan, K., 1998. With distributors in mind, Northwestern embraces VMI. Metal Center News 38 (3), 32–36.

Schenck, J., McInerney, J., 1998. Applying vendor-managed inventory to the apparel industry. Automatic I.D. News 14 (6), 36–38.

Speed, K.K., 1998. Finding ways to earn business. Industrial Distribution 87 (3), S6–S8.

Thomas, D.J., Griffin, P.M., 1996. Coordinated supply chain management. European Journal of Operational Research 94, 1–15.

Waller, M., Johnson, M.E., Davis, T., 1999. Vendor-managed inventory in the retail supply chain. Journal of Business Logistics 20 (1), 183–203.

Weng, Z.K., 1995. Channel coordination and quantity discount. Management Science 41 (9), 1509–1522.

Xu, K., Dong, Y., Evers, P.T., 2001. Towards better coordination of the supply chain. Transportation Research Part E: Logistics and Transportation Review 37 (1), 35–54.